黑体辐射——瑞利金斯公式

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02

Formula derivation

《Remarks upon the Law of Complete Radiation》

Wien concludes that the actual law is

$$c_1 \lambda^{-5} e^{-c_2/\lambda\theta} d\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which c_1 and c_2 are constants, but viewed from the theoretical side the result appears to me to be little more than a conjecture. It is, however, supported upon general thermodynamic grounds by Planck ‖.

Upon the experimental side, Wien's law (2) has met with important confirmation. Paschen finds that his observations are well represented, if he takes

$$c_2 = 14{,}455,$$

θ being measured in centigrade degrees and λ in thousandths of a millimetre (μ). Nevertheless, the law seems rather difficult of acceptance, especially the implication that as the temperature is raised, the radiation of given wave-length approaches a limit. It is true that for visible rays the limit is out of range. But if we take $\lambda = 60\mu$, as (according to the remarkable researches of Rubens) for the rays selected by reflexion at surfaces of Sylvin, we see that for temperatures over 1000° (absolute) there would be but little further increase of radiation.

The question is one to be settled by experiment; but in the meantime I venture to suggest a modification of (2), which appears to me more probable *à priori*. Speculation upon this subject is hampered by the difficulties which attend the Boltzmann-Maxwell doctrine of the partition of energy. According to this doctrine every mode of vibration should be alike favoured; and although for some reason not yet explained the doctrine fails in general, it seems possible that it may apply to the graver modes. Let us consider in illustra-

《The Dynamical Theory of Gases and of Radiation》

As an introduction, we consider the motion of a stretched string of length l, vibrating transversely in one plane. If a be the velocity of propagation, ξ the number of subdivisions in any mode of vibration, the frequency f is given by

$$f = a\xi/2l \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

A passage from any mode to the next in order involves a change of unity in the value of ξ, or of $2lf/a$. Hence if e denote the kinetic energy of a single mode, the law of equipartition requires that the kinetic energy corresponding to the interval df shall be

$$2le/a.df \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

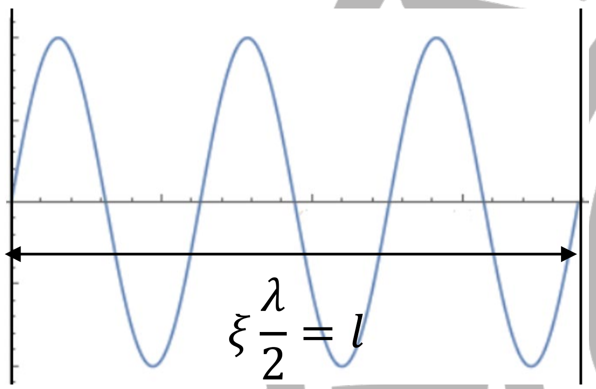
In terms of λ the wave-length, (2) becomes

$$2le/\lambda^2.d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

This is for the whole length of the string. The longitudinal *density* of the kinetic energy is accordingly

$$2e/\lambda^2.d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

考虑长度为l的弦的横振动（约束在一个平面内）



计算df区间的振动模式数目，得到

$$df = \frac{a}{2l}d\xi$$

用e表示一个振动模式的动能，由$dE_k = ed\xi$，得到

$$dE_k = \frac{2le}{a}df$$

In each mode the potential energy is (on the average) equal to the kinetic, so that if we wish to reckon the whole energy, (4) must be doubled. Another doubling ensues when we abandon the restriction to one plane of vibration; and finally for the total energy corresponding to the interval from λ to $\lambda + d\lambda$ we have

$$8e/\lambda^2 . d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

When we proceed to three dimensions, and consider the vibrations within a cube of side l, subdivisions may occur in three directions. In place of (1)

$$f = a/2l . \sqrt{(\xi^2+\eta^2+\zeta^2)} \quad . \quad . \quad . \quad . \qquad (6)$$

where ξ, η, ζ may assume any integral values. The next step is to ascertain what is the number of modes which corresponds to an assigned variation of f.

If the integral values of ξ, η, ζ be regarded as the coordinates of a point, the whole system of points constitutes a cubic array of volume-density unity. If R be the distance of any point from the origin,

$$R^2 = \xi^2+\eta^2+\zeta^2 ;$$

and the number of points between R and R+dR, equal to the included volume, is

$$4\pi R^2 dR.$$

Hence the number of modes corresponding to df is

$$4\pi(2l/a)^3 f^2 df,$$

or in terms of λ

$$4\pi . 8l^3 . \lambda^{-4} d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

If e be the kinetic energy in each mode, then the kinetic energy corresponding to $d\lambda$ and to unit of volume is

$$32\pi e\lambda^{-4} d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Since, as in the case of the string, we are dealing with transverse vibrations, and since the whole energy is the double of the kinetic energy, we have finally

$$\boxed{128 . \pi . e . \lambda^{-4} d\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

$$e = \frac{1}{2}KT$$

as the total energy of radiation per unit of volume corresponding to the interval from λ to λ + dλ, and in (9) e is proportional to the absolute temperature θ.

Apart from the numerical coefficient, this is the formula which I gave in the paper referred to as probably representing the truth when λ is large, in place of the quite different form then generally accepted. The suggestion was soon confirmed by Rubens and Kurlbaum, and a little later

$$E\mathrm{d}\lambda = 64\pi k\theta\lambda^{-4}d\lambda$$

Planck (*Drude Ann.*, vol. iv. p. 553, 1901) put forward his theoretical formula, which seems to agree very well with the experimental facts. This contains two constants, h and k, besides c, the velocity of light. In terms of λ it is

$$E\,d\lambda = \frac{8\pi ch}{\lambda^5}\,\frac{d\lambda}{e^{ch/k\lambda\theta}-1} \quad . \quad . \quad . \quad (10)$$

reducing when λ is great to

$$E\,d\lambda = 8\pi k\theta\lambda^{-4}d\lambda \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in agreement with (9). $E\,d\lambda$ here denotes the volume-density of the energy of radiation corresponding to $d\lambda$.

θ being measured in centigrade degrees. This result is eight times as large as that found by Planck. If we re-

《The Constant of Radiation as Calculated from Molecular Date》

In NATURE, May 18, I gave a calculation of the coefficient of complete radiation at a given absolute temperature for waves of great length on principles laid down in 1900, and it appeared that the result was eight times as great as that deduced from Planck's formula for this case. In connection with similar work of his own, Mr. Jeans (*Phil. Mag.*, July) has just pointed out that I have introduced a redundant factor 8 by counting negative as well as positive values of my integers ξ, η, ζ.

03

Supplement

《The Dynamical Theory of Gases and of Radiation》

According to (15), if it were applicable to all wave-lengths, the total energy of radiation at a given temperature would be infinite, and this is an inevitable consequence of applying the law of equipartition to a uniform structureless medium. If we were dealing with elastic solid balls

Can we escape from the difficulties, into which we have been led, by appealing to the slowness with which equipartition may establish itself? According to this view, the energy of radiation within an enclosure at given temperature would, indeed, increase without limit, but the rate of increase after a short time would be very slow. If a

It seems to me that we must admit the failure of the law of equipartition in these extreme cases. If this is so, it is obviously of great importance to ascertain the reason.

《The Dynamical Theory of Gases and of Radiation》

May I, in the first place, suggest that the slowness with which energy is transferred to the quicker modes of ether-vibration is a matter of calculation, and not of speculation? If the average time of collision of two molecules in a gas is a great multiple N of the period of a vibration, whether of matter or of ether, then the average transfer of energy to the vibration per collision can be shown to contain a factor of the order of smallness of e^{-N}. The calculations will be found in §§ 236–244 of

The factor e^{-N} is so small for most of the ether-vibrations as to be negligible. There is no sharp line of demarcation between those vibrations which acquire energy very slowly and those for which the rate is appreciable; but as e^{-N} varies rapidly with N when N is large, there will be but few vibrations near the border, so that it seems legitimate, for purposes of a general discussion, to divide the vibrations into the two distinct classes, quick and slow, relatively to the scale of time provided by molecular collisions.

这部分振动模式达到能量均分定理要求的能量所需时间极长，所以这部分振动模式没有可观的能量

振动周期小于平均碰撞时间的振动模式

每一次分子碰撞导致的能量传递极小

振动周期大于平均碰撞时间的振动模式

通过分子碰撞能很快分配到能量均分定理要求的能量

这部分振动模式的数目总是有限的

04

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谢谢

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